Chapter 12

PROJECTIONS OF PLANES

12-0. INTRODUCTION

Plane figures or surfaces have only two dimensions, viz. length and breadth. They do not have thickness. A plane figure may be assumed to be contained by a plane, and its projections can be drawn, if the position of that plane with respect to the principal planes of projection is known.

In this chapter, we shall discuss the following topics:

1. Types of planes and their projections.
2. Traces of planes.

12-1. TYPES OF PLANES

Planes may be divided into two main types:

(1) Perpendicular planes.

(2) Oblique planes.

(1) Perpendicular planes: These planes can be divided into the following sub-types:

(i) Perpendicular to both the reference planes.

(ii) Perpendicular to one plane and parallel to the other.

(iii) Perpendicular to one plane and inclined to the other.

(i) *Perpendicular to both the reference planes* (fig. 12-1): A square *ABCD* is perpendicular to both the planes. Its H.T. and V.T. are in a straight line perpendicular to *xy*.

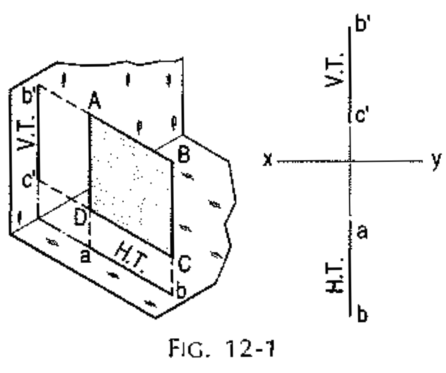


FIG. 12-1

The front view *b'c'* and the top view *ab* of the square are both lines coinciding with the V.T. and the H.T. respectively.

(ii) ***Perpendicular to one plane and parallel to the other plane:***

(a) Plane, perpendicular to the H.P. and parallel to the V.P. [fig. 12-2(i)].

A triangle *PQR* is perpendicular to the H.P. and is parallel to the V.P. Its H.T. is parallel to *xy*. It has no V.T.

The front view *p'q'r'* shows the exact shape and size of the triangle. The top view *pqr* is a line parallel to *xy*. It coincides with the H.T.

(b) Plane, perpendicular to the V.P. and parallel to the H.P. [fig. 12-2(ii)].

A square *ABCD* is perpendicular to the V.P. and parallel to the H.P. Its V.T. is parallel to *xy*. It has no H.T.

The top view *abcd* shows the true shape and true size of the square. The front view *a'b'* is a line, parallel to *xy*. It coincides with the V.T.

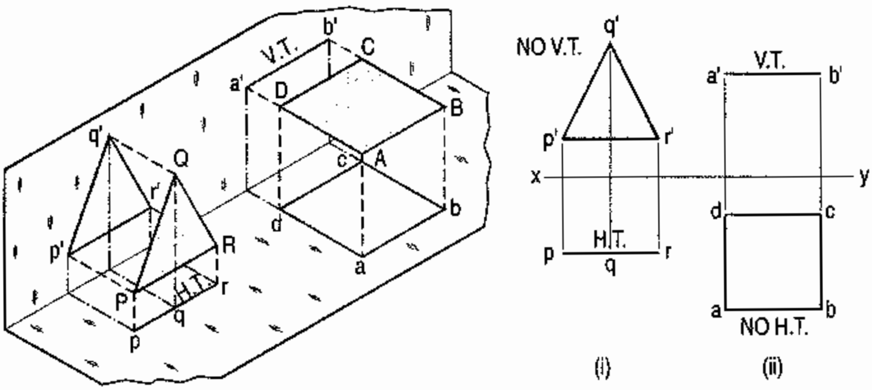


FIG. 12-2

(iii) ***Perpendicular to one plane and inclined to the other plane:***

(a) Plane, perpendicular to the H.P. and inclined to the V.P. (fig. 12-3).

A square *ABCD* is perpendicular to the H.P. and inclined at an angle ø to the V.P. Its V.T. is perpendicular to *xy*. Its H.T. is inclined at ø to xy.

Its top view *ab* is a line linclined at ø to *xy*. The front view *a'b'c'd'* is smaller than *ABCD*.

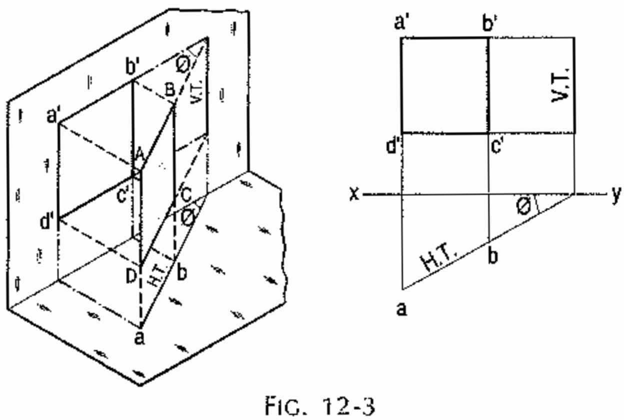


FIG. 12-3

(b) Plane, perpendicular to the V.P. and inclined to the H.P. (fig. 12-4).

A square *ABCD* is perpendicular to the V.P. and inclined at an angle θ to the H.P. Its H.T. is perpendicular to *xy*. Its V.T. makes the angle θ with *xy*. Its front view *a'b'* is a line inclined at θ to *xy*. The top view *abcd* is a rectangle which is smaller than the square *ABCD*.

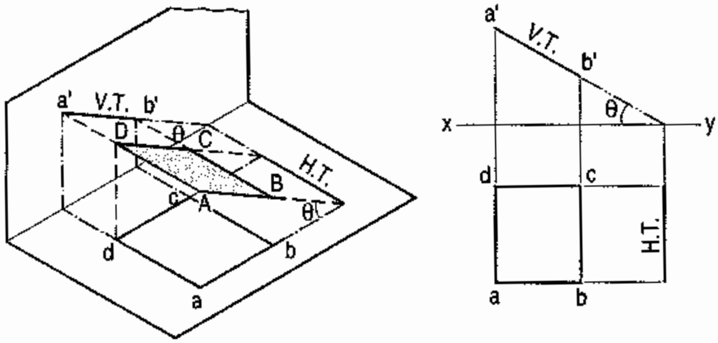


FIG. 12-4

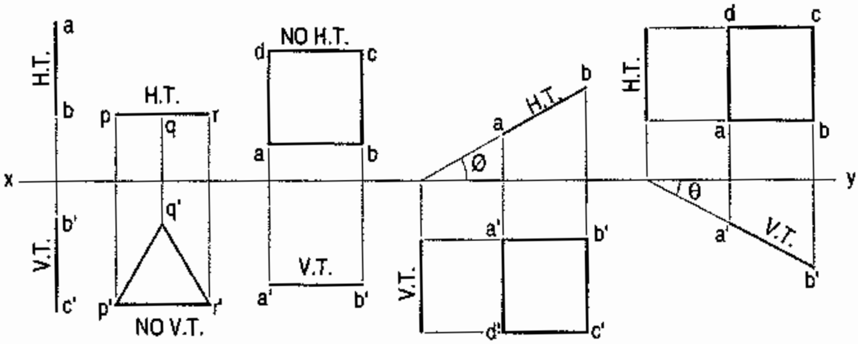


FIG. 12-5

Fig. 12-5 shows the projections and the traces of all these perpendicular planes by third-angle projection method.

(2) Oblique planes: Planes which are inclined to both the reference planes are called *oblique planes*. Representation of oblique planes by their traces is too advanced to be included in this book.

A few problems on the projections of plane figures inclined to both the reference planes are however, illustrated at the end of the chapter. They will prove to be of great use in dealing with the projections of solids.

12-2. TRACES OF PLANES

A plane, extended if necessary, will meet the reference planes in lines, unless it is parallel to any one of them.

These lines are called the *traces* of the plane. The line in which the plane meets the H.P. is called the *horizontal trace* or the H.T. of the plane. The line in which it meets the V.P. is called its *vertical trace* or the V.T. A plane is usually represented by its traces.

12-3. GENERAL CONCLUSIONS

(1) Traces:

(a) When a plane is perpendicular to both the reference planes, its traces lie on a straight line perpendicular to *xy*.

(b) When a plane is perpendicular to one of the reference planes, its trace upon the other plane is perpendicular to *xy* (except when it is parallel to the other plane).

(c) When a plane is parallel to a reference plane, it has no trace on that plane. Its trace on the other reference plane, to which it is perpendicular, is parallel to *xy*.

(d) When a plane is inclined to the H.P. and perpendicular to the V.P., its inclination is shown by the angle which its V.T. makes with *xy*. When it is inclined to the V.P. and perpendicular to the H.P., its inclination is shown by the angle which its H.T. makes with *xy*.

(e) When a plane has two traces, they, produced if necessary, intersect in *xy* (except when both are parallel to *xy* as in case of some oblique planes).

(2) Projections:

(a) When a plane is perpendicular to a reference plane, its projection on that plane is a straight line.

(b) When a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

(c) When a plane is perpendicular to one of the reference planes and inclined to the other, its inclination is shown by the angle which its projection on the plane to which it is perpendicular, makes with *xy*. Its projection on the plane to which it is inclined, is smaller than the plane itself.

Problem 12-1. *Show by means of traces, each of the following planes:*

(a) *Perpendicular to the H.P. and the V.P.*

(b) *Perpendicular to the H.P. and inclined at 30° to the V.P.*

(c) *Parallel to and 40 mm away from the V.P.*

(d) *Inclined at 45° to the H.P. and perpendicular to the V.P.*

(e) *Parallel to the H.P. and 25 mm away from it.*

Fig. 12-6 and fig. 12-7 show the various traces.

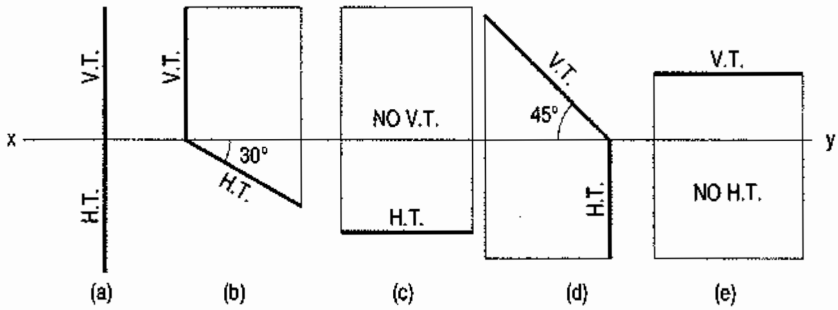
(a) The H.T. and the V.T. are in a line perpendicular to xy.

(b) The H.T. is inclined at 30° to *xy*; the V.T. is normal to *xy*; both the traces intersect in *xy*.

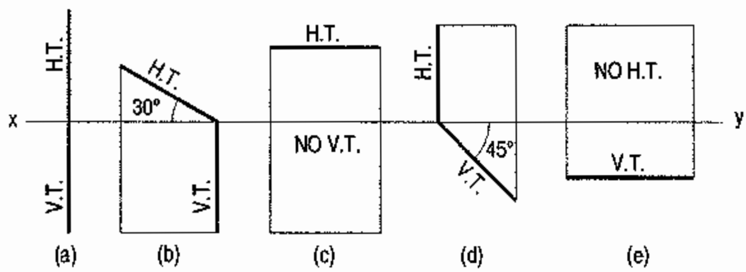
(c) The H.T. is parallel to and 40 mm away from *xy*. It has no V.T.

(d) The H.T. is perpendicular to *xy*; the V.T. makes 45° angle with *xy*; both intersect in *xy*.

(e) The V.T. is parallel to and 25 mm away from *xy*. It has no H.T.



(First-angle projection)
FIG. 12-6



(Third-angle projection)
FIG. 12-7

12-4. PROJECTIONS OF PLANES PARALLEL TO ONE OF THE REFERENCE PLANES

The projection of a plane on the reference plane parallel to it will show its true shape. Hence, beginning should be made by drawing that view. The other view which will be a line, should then be projected from it.

(1) When the plane is parallel to the H.P.: The top view should be drawn first and the front view projected from it.

Problem 12-2. (fig. 12-8): *An equilateral triangle of 50 mm side has its V.T. parallel to and 25 mm above xy. It has no H.T. Draw its projections when one of its sides is inclined at 45° to the V.P.*

As the V.T. is parallel to *xy* and as there is no H.T. the triangle is parallel to the H.P. Therefore, begin with the top view.

(i) Draw an equilateral triangle *abc* of 50 mm side, keeping one side, say *ac*, inclined at 45° to *xy*.

(ii) Project the front view, parallel to and 25 mm *above* *xy*, as shown.

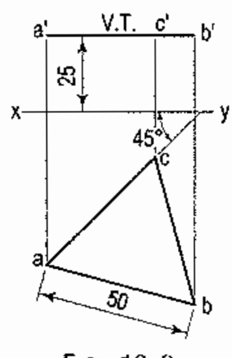


FIG. 12-8

(2) When the plane is parallel to the V.P.: Beginning should be made with the front view and the top view projected from it.

Problem 12-3. (fig. 12-9): *A square ABCD of 40 mm side has a corner on the H.P. and 20 mm in front of the V.P. All the sides of the square are equally inclined to the H.P. and parallel to the V.P. Draw its projections and show its traces.*

As all the sides are parallel to the V.P., the surface of the square also is parallel to it. The front view will show the true shape and position of the square.

(i) Draw a square *a'b'c'd'* in the front view with one corner in *xy* and all its sides inclined at 45° to *xy*.

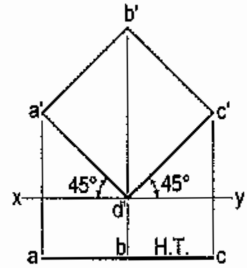


FIG. 12-9

(ii) Project the top view keeping the line *ac* parallel to *xy* and 30 mm below it. The top view is its H.T. It has no V.T.

12-5. PROJECTIONS OF PLANES INCLINED TO ONE REFERENCE PLANE AND PERPENDICULAR TO THE OTHER

When a plane is inclined to a reference plane, its projections may be obtained in two stages. In the initial stage, the plane is assumed to be parallel to that reference plane to which it has to be made inclined. It is then tilted to the required inclination in the second stage.

(1) Plane, inclined to the H.P. and perpendicular to the V.P.: When the plane is inclined to the H.P. and perpendicular to the V.P., in the initial stage, it is assumed to be parallel to the H.P. Its top view will show the true shape. The front view will be a line parallel to *xy*. The plane is then tilted so that it is inclined to the H.P. The new front view will be inclined to *xy* at the true inclination. In the top view the corners will move along their respective paths (parallel to *xy*).

Problem 12-4. (fig. 12-10): *A regular pentagon of 25 mm side has one side on the ground. Its plane is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections and show its traces.*

Assuming it to be parallel to the H.P.

(i) Draw the pentagon in the top view with one side perpendicular to *xy* [fig. 12-10(i)]. Project the front view. It will be the line *a'c'* contained by *xy*.

(ii) Tilt the front view about the point a', so that it makes 45° angle with *xy*.

(iii) Project the new top view $ab_1c_1d_1e$ upwards from this front view and horizontally from the first top view. It will be more convenient if the front view is reproduced in the new position separately and the top view projected from it, as shown in fig. 12-10(ii). The V.T. coincides with the front view and the H.T. is perpendicular to *xy*, through the point of intersection between *xy* and the front view-produced.

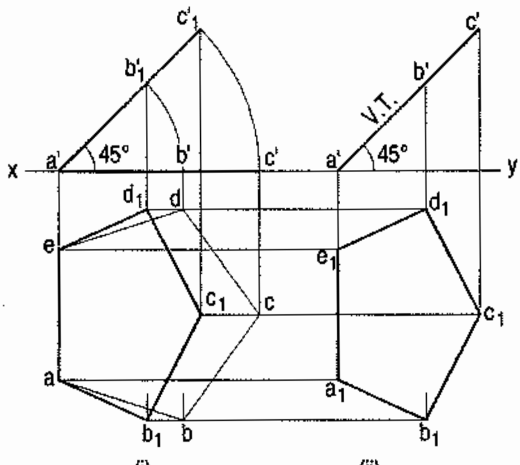


FIG. 12-10

(2) Plane, inclined to the V.P. and perpendicular to the H.P.: In the initial stage, the plane may be assumed to be parallel to the V.P. and then tilted to the required position in the next stage. The projections are drawn as illustrated in the next problem.

Problem 12-5. (fig. 12-11): *Draw the projections of a circle of 50 mm diameter, having its plane vertical and inclined at 30° to the V.P. Its centre is 30 mm above the H.P. and 20 mm in front of the V.P. Show also its traces.*

A circle has no corners to project one view from another. However, a number of points, say twelve, equal distances apart, may be marked on its circumference.

(i) Assuming the circle to be parallel to the V.P., draw its projections. The front view will be a circle [fig. 12-11(i)], having its centre 30 mm above *xy*. The top view will be a line, parallel to and 20 mm below *xy*.

(ii) Divide the circumference into twelve equal parts (with a 30°-60° set-square) and mark the points as shown. Project these points in the top view. The centre *O* will coincide with the point 4.

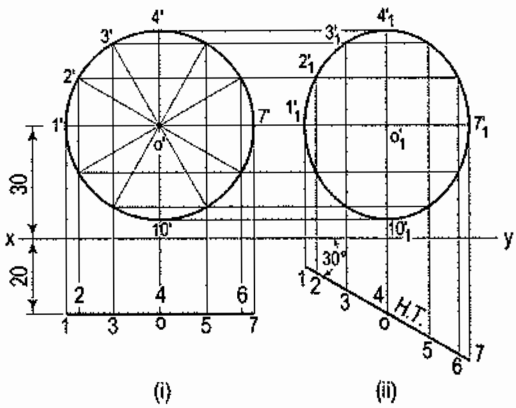


FIG. 12-11

(iii) When the circle is tilted, so as to make 30° angle with the V.P., its top view will become inclined at 30° to *xy*. In the front view all the points will move along their respective paths (parallel to *xy*). Reproduce the top view keeping the centre *o* at the same distance, viz. 20 mm from *xy* and inclined at 30° to *xy* [fig. 12-11(ii)].

(iv) For the final front view, project all the points upwards from this top view and horizontally from the first front view. Draw a freehand curve through the twelve points $1'_1$, $2'_1$ etc. This curve will be an ellipse.

12-6. PROJECTIONS OF OBLIQUE PLANES

When a plane has its surface inclined to one plane and an edge or a diameter or a diagonal parallel to that plane and inclined to the other plane, its projections are drawn in three stages.

(1) If the surface of the plane is inclined to the H.P. and an edge (or a diameter or a diagonal) is parallel to the H.P. and inclined to the V.P.,

   (i) in the initial position the plane is assumed to be parallel to the H.P. and an edge perpendicular to the V.P.

   (ii) It is then tilted so as to make the required angle with the H.P. As already explained, its front view in this position will be a line, while its top view will be smaller in size.

   (iii) In the final position, when the plane is turned to the required inclination with the V.P., only the position of the top view will change. Its shape and size will not be affected. In the final front view, the corresponding distances of all the corners from *xy* will remain the same as in the second front view.

Chapter 13

PROJECTIONS OF SOLIDS

13-0. INTRODUCTION

A solid has three dimensions, viz. length, breadth and thickness. To represent a solid on a flat surface having only length and breadth, at least two orthographic views are necessary. Sometimes, additional views projected on auxiliary planes become necessary to make the description of a solid complete.

This chapter deals with the following topics:

1. Types of solids.
2. Projections of solids in simple positions.
   - (a) Axis perpendicular to the H.P.
   - (b) Axis perpendicular to the V.P.
   - (c) Axis parallel to both the H.P. and the V.P.
3. Projections of solids with axes inclined to one of the reference planes and parallel to the other.
   - (a) Axis inclined to the V.P. and parallel to the H.P.
   - (b) Axis inclined to the H.P. and parallel to the V.P.
4. Projections of solids with axes inclined to both the H.P. and the V.P.
5. Projections of spheres.

13-1. TYPES OF SOLIDS

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 28 for the types of solids.

Solids may be divided into *two* main groups:

(1) Polyhedra

(2) Solids of revolution.

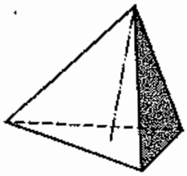
(1) Polyhedra: A polyhedron is defined as a solid bounded by planes called *faces*. When all the faces are equal and regular, the polyhedron is said to be regular.

There are *seven* regular polyhedra which may be defined as stated below:

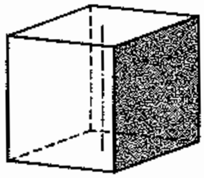
(i) *Tetrahedron* (fig. 13-1): It has four equal faces, each an equilateral triangle.

(ii) *Cube or hexahedron* (fig. 13-2): It has six faces, all equal squares.

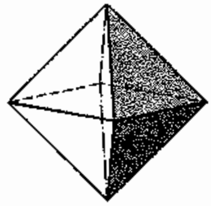
(iii) *Octahedron* (fig. 13-3): It has eight equal equilateral triangles as faces.



Tetrahedron
FIG. 13-1



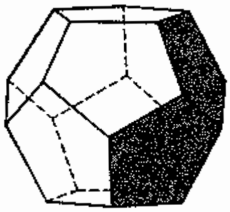
Cube
FIG. 13-2



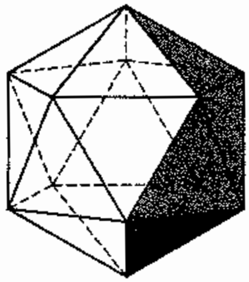
Octahedron
FIG. 13-3

(iv) *Dodecahedron* (fig. 13-4): It has twelve equal and regular pentagons as faces.

(v) *Icosahedron* (fig. 13-5): It has twenty faces, all equal equilateral triangles.



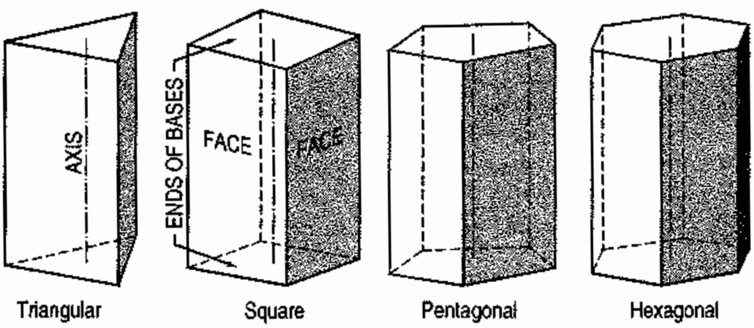
Dodecahedron
FIG. 13-4



Icosahedron
FIG. 13-5

(vi) *Prism:* This is a polyhedron having two equal and similar faces called its ends or bases, parallel to each other and joined by other faces which are parallelograms. The imaginary line joining the centres of the bases is called the axis.

A right and regular prism (fig. 13-6) has its axis perpendicular to the bases. All its faces are equal rectangles.



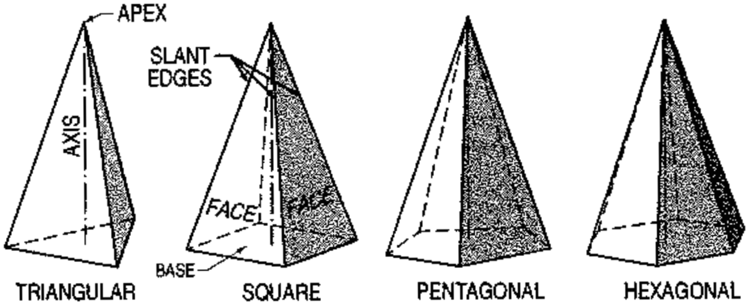
Prisms
FIG. 13-6

(vii) *Pyramid:* This is a polyhedron having a plane figure as a base and a number of triangular faces meeting at a point called the vertex or apex. The imaginary line joining the apex with the centre of the base is its axis.

A right and regular pyramid (fig. 13-7) has its axis perpendicular to the base which is a regular plane figure. Its faces are all equal isosceles triangles.

Oblique prisms and pyramids have their axes inclined to their bases.

Prisms and pyramids are named according to the shape of their bases, as triangular, square, pentagonal, hexagonal etc.



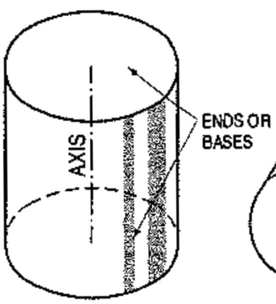
Pyramids
FIG. 13-7

(2) Solids of revolution:

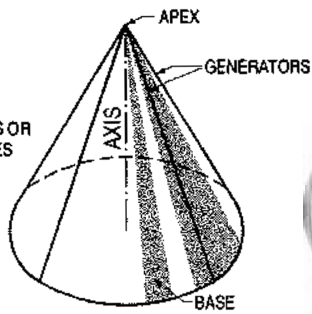
(i) *Cylinder* (fig. 13-8): A *right circular cylinder* is a solid generated by the revolution of a rectangle about one of its sides which remains fixed. It has two equal circular bases. The line joining the centres of the bases is the axis. It is perpendicular to the bases.

(ii) *Cone* (fig. 13-9): A *right circular cone* is a solid generated by the revolution of a right-angled triangle about one of its perpendicular sides which is fixed.

It has one circular base. Its axis joins the apex with the centre of the base to which it is perpendicular. Straight lines drawn from the apex to the circumference of the base-circle are all equal and are called *generators* of the cone. The length of the generator is the slant height of the cone.



Cylinder
FIG. 13-8



Cone
FIG. 13-9

Sphere
FIG. 13-10

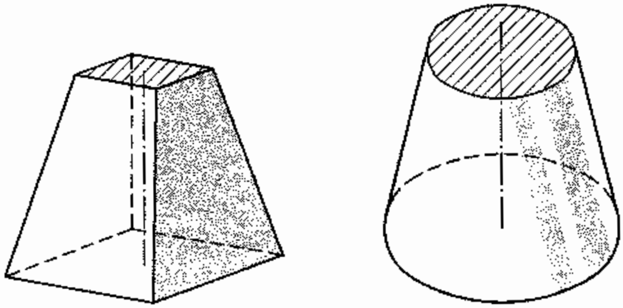
(iii) *Sphere* (fig. 13-10): A *sphere* is a solid generated by the revolution of a semi-circle about its diameter as the axis. The mid-point of the diameter is the centre of the sphere. All points on the surface of the sphere are equidistant from its centre.

Oblique cylinders and cones have their axes inclined to their bases.

(iv) *Frustum*: When a pyramid or a cone is cut by a plane parallel to its base, thus removing the top portion, the remaining portion is called its *frustum* (fig. 13-11).

(v) *Truncated*: When a solid is cut by a plane inclined to the base it is said to be *truncated*.

In this book mostly right and regular solids are dealt with. Hence, when a solid is named without any qualification, it should be understood as being right and regular.



Frustums
FIG. 13-11

13-2. PROJECTIONS OF SOLIDS IN SIMPLE POSITIONS

A solid in simple position may have its axis perpendicular to one reference plane or parallel to both. When the axis is perpendicular to one reference plane, it is parallel to the other. Also, when the axis of a solid is perpendicular to a plane, its base will be parallel to that plane. We have already seen that when a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

Therefore, the projection of a solid on the plane to which its axis is perpendicular, will show the true shape and size of its base.

Hence, when the axis is perpendicular to the ground, i.e. to the H.P., the top view should be drawn first and the front view projected from it.

When the axis is perpendicular to the V.P., beginning should be made with the front view. The top view should then be projected from it.

When the axis is parallel to both the H.P. and the V.P., neither the top view nor the front view will show the actual shape of the base. In this case, the projection of the solid on an auxiliary plane perpendicular to both the planes, viz. the side view must be drawn first. The front view and the top view are then projected from the side view. The projections in such cases may also be drawn in two stages.

(1) Axis perpendicular to the H.P.:

Problem 13-1. (fig. 13-12): *Draw the projections of a triangular prism, base 40 mm side and axis 50 mm long, resting on one of its bases on the H.P. with a vertical face perpendicular to the V.P.*

(i) As the axis is perpendicular to the ground i.e. the H.P. begin with the top view. It will be an equilateral triangle of sides 40 mm long, with one of its

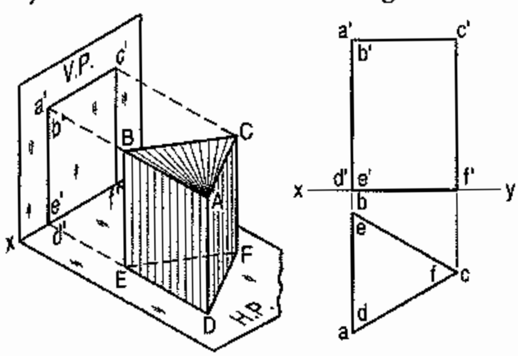


FIG. 13-12

sides perpendicular to xy. Name the corners as shown, thus completing the top view. The corners d, e and f are hidden and coincide with the top corners a, b and c respectively.

(ii) Project the front view, which will be a rectangle. Name the corners. The line $b'e'$ coincides with $a'd'$.

Problem 13-2. (fig. 13-13): *Draw the projections of a pentagonal pyramid, base 30 mm edge and axis 50 mm long, having its base on the H.P. and an edge of the base parallel to the V.P. Also draw its side view.*

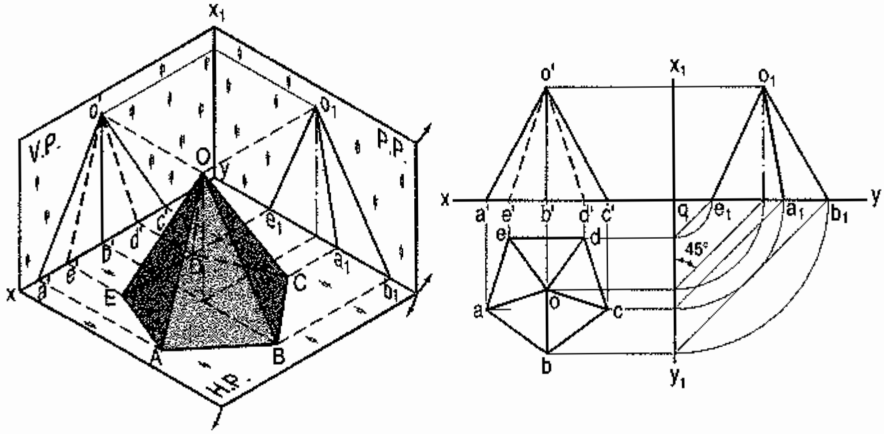


FIG. 13-13

(i) Assume the side DE which is nearer the V.P., to be parallel to the V.P. as shown in the pictorial view.

(ii) In the top view, draw a regular pentagon $abcde$ with ed parallel to and nearer xy. Locate its centre o and join it with the corners to indicate the slant edges.

(iii) Through o, project the axis in the front view and mark the apex o', 50 mm above xy. Project all the corners of the base on xy. Draw lines $o'a'$, $o'b'$ and $o'c'$ to show the visible edges. Show $o'd'$ and $o'e'$ for the hidden edges as dashed lines.

(iv) For the side view looking from the left, draw a new reference line x_1y_1 perpendicular to xy and to the right of the front view. Project the side view on it, horizontally from the front view as shown. The respective distances of all the points in the side view from x_1y_1, should be equal to their distances in the top view from xy. This is done systematically as explained below:

(v) From each point in the top view, draw horizontal lines upto x_1y_1. Then draw lines inclined at 45° to x_1y_1 (or xy) as shown. Or, with q, the point of intersection between xy and x_1y_1 as centre, draw quarter circles. Project up all the points to intersect the corresponding horizontal lines from the front view and complete the side view as shown in the figure. Lines o_1d_1 and o_1c_1 coincide with o_1e_1 and o_1a_1 respectively.

Problem 13-3. (fig. 13-14): *Draw the projections of (i) a cylinder, base 40 mm diameter and axis 50 mm long, and (ii) a cone, base 40 mm diameter and axis 50 mm long, resting on the H.P. on their respective bases.*

(i) Draw a circle of 40 mm diameter in the top view and project the front view which will be a rectangle [fig. 13-14(ii)].

(ii) Draw the top view [fig. 13-14(iii)]. Through the centre o, project the apex o', 50 mm above *xy*. Complete the triangle in the front view as shown.

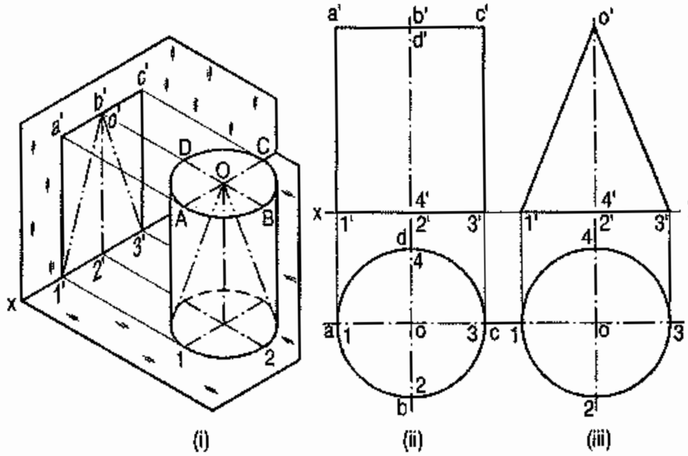


FIG. 13-14

In the pictorial view [fig. 13-14(i)], the cone is shown as contained by the cylinder.

Problem 13-4. (fig. 13-15): *A cube of 50 mm long edges is resting on the H.P. with its vertical faces equally inclined to the V.P. Draw its projections.*

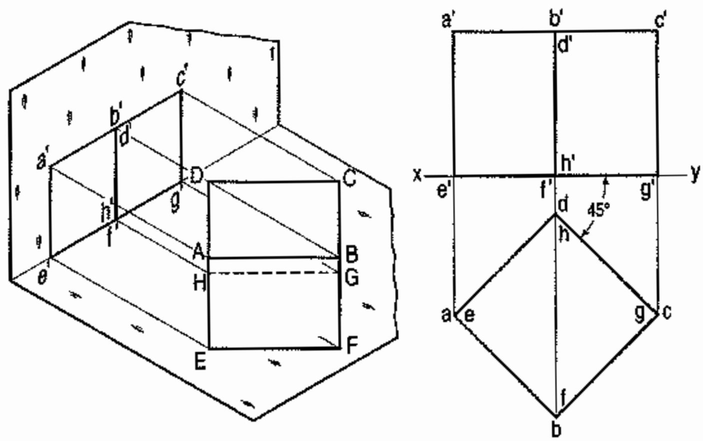


FIG. 13-15

Begin with the top view.

(i) Draw a square *abcd* with a side making 45° angle with *xy*.

(ii) Project up the front view. The line *d' h'* will coincide with *b' f'*.

Problem 13-5. (fig. 13-16): *Draw the projections of a hexagonal pyramid, base 30 mm side and axis 60 mm long, having its base on the H.P. and one of the edges of the base inclined at 45° to the V.P.*

(i) In the top view, draw a line *af* 30 mm long and inclined at 45° to *xy*. Construct a regular hexagon on *af*. Mark its centre o and complete the top view by drawing lines joining it with the corners.

(ii) Project up the front view as described in problem 13-2, showing the line *o'e'* and *o'f'* for hidden edges as dashed lines.

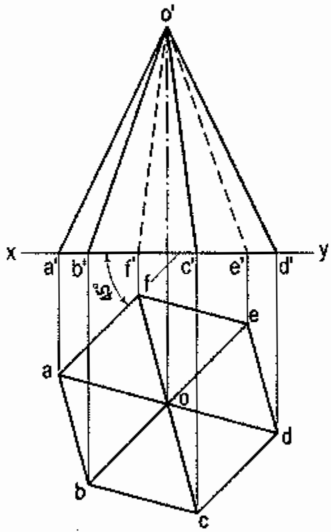


FIG. 13-16

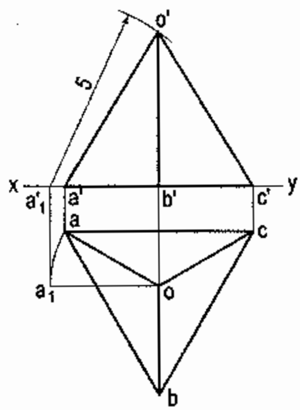


FIG. 13-17

Problem 13-6. (fig. 13-17): *A tetrahedron of 5 cm long edges is resting on the H.P. on one of its faces, with an edge of that face parallel to the V.P. Draw its projections and measure the distance of its apex from the ground.*

All the four faces of the tetrahedron are equal equilateral triangles of 5 cm side.

(i) Draw an equilateral triangle *abc* in the top view with one side, say *ac*, parallel to *xy*. Locate its centre o and join it with the corners.

(ii) In the front view, the corners *a'*, *b'* and *c'* will be in *xy*. The apex *o'* will lie on the projector through *o* so that its true distance from the corners of the base is equal to 5 cm.

(iii) To locate o', make *oa* (or *ob* or *oc*) parallel to *xy*. Project a_1 to a'_1 on *xy*. With a'_1 as centre and radius equal to 5 cm cut the projector through *o* in *o'*. Draw lines *o'a'*, *o'b'* and *o'c'* to complete the front view. *o'b'* will be the distance of the apex from the ground.

(2) Axis perpendicular to the V.P.:

Problem 13-7. (fig. 13-18): *A hexagonal prism has one of its rectangular faces parallel to the H.P. Its axis is perpendicular to the V.P. and 3.5 cm above the ground.*

Draw its projections when the nearer end is 2 cm in front of the V.P. Side of base 2.5 cm long; axis 5 cm long.

(i) Begin with the front view. Construct a regular hexagon of 2.5 cm long sides with its centre 3.5 cm above *xy* and one side parallel to it.

(ii) Project down the top view, keeping the line for nearer end, viz. 1-4, 2 cm below *xy*.

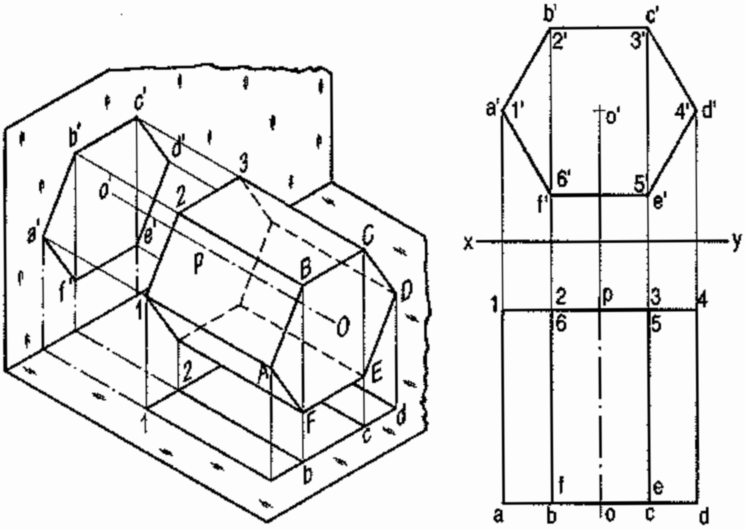


FIG. 13-18

Problem 13-8. (fig. 13-19): *A square pyramid, base 40 mm side and axis 65 mm long, has its base in the V.P. One edge of the base is inclined at 30° to the H.P. and a corner contained by that edge is on the H.P. Draw its projections.*

(i) Draw a square in the front view with the corner d' in xy and the side $d'c'$ inclined at 30° to it. Locate the centre o' and join it with the corners of the square.

(ii) Project down all the corners in xy (because the base is in the V.P.). Mark the apex o on a projector through o'. Draw lines for the slant edges and complete the top view.

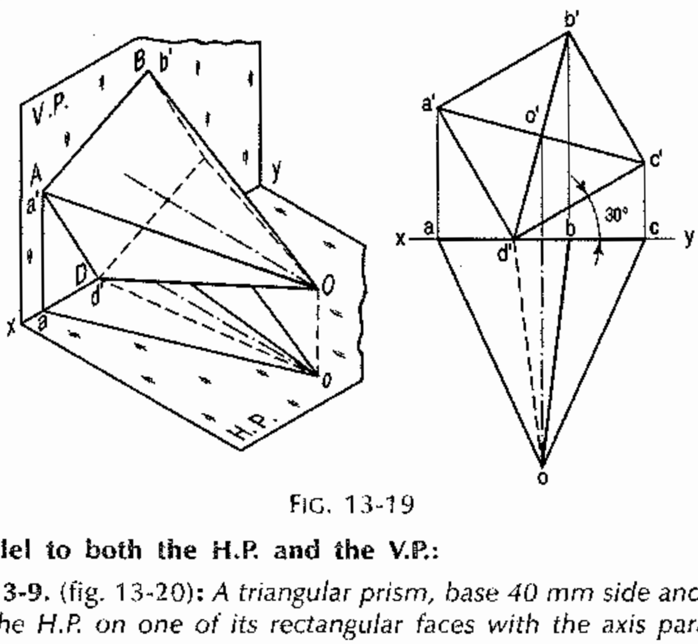


FIG. 13-19

(3) Axis parallel to both the H.P. and the V.P.:

Problem 13-9. (fig. 13-20): *A triangular prism, base 40 mm side and height 65 mm is resting on the H.P. on one of its rectangular faces with the axis parallel to the V.P. Draw its projections.*

As the axis is parallel to both the planes, begin with the side view.

(i) Draw an equilateral triangle representing the side view, with one side in *xy*.

(ii) Project the front view horizontally from this triangle.

(iii) Project down the top view from the front view and the side view, as shown.

This problem can also be solved in two stages as explained in the next article.

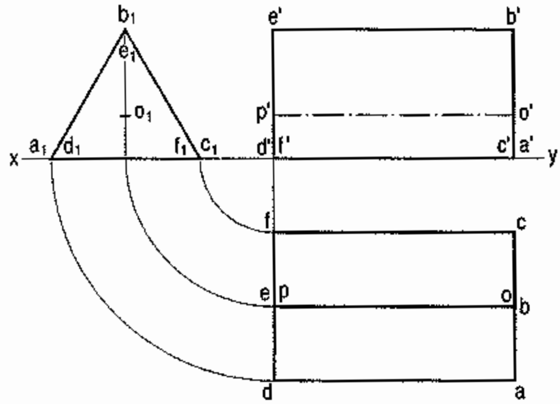


FIG. 13-20

EXERCISES 13(a)

Draw the projections of the following solids, situated in their respective positions, taking a side of the base 40 mm long or the diameter of the base 50 mm long and the axis 65 mm long.

1. A hexagonal pyramid, base on the H.P. and a side of the base parallel to and 25 mm in front of the V.P.
2. A square prism, base on the H.P., a side of the base inclined at 30° to the V.P. and the axis 50 mm in front of the V.P.
3. A triangular pyramid, base on the H.P. and an edge of the base inclined at 45° to the V.P.; the apex 40 mm in front of the V.P.
4. A cylinder, axis perpendicular to the V.P. and 40 mm above the H.P., one end 20 mm in front of the V.P.
5. A pentagonal prism, a rectangular face parallel to and 10 mm above the H.P., axis perpendicular to the V.P. and one base in the V.P.
6. A square pyramid, all edges of the base equally inclined to the H.P. and the axis parallel to and 50 mm away from both the H.P. and the V.P.
7. A cone, apex in the H.P. axis vertical and 40 mm in front of the V.P.
8. A pentagonal pyramid, base in the V.P. and an edge of the base in the H.P.

13-3. PROJECTIONS OF SOLIDS WITH AXES INCLINED TO ONE OF THE REFERENCE PLANES AND PARALLEL TO THE OTHER

When a solid has its axis inclined to one plane and parallel to the other, its projections are drawn in two stages.

(1) In the initial stage, the solid is assumed to be in simple position, i.e. *its axis perpendicular to one of the planes.*

If the axis is to be inclined to the ground, i.e. the H.P., it is assumed to be perpendicular to the H.P. in the initial stage. Similarly, if the axis is to be inclined to the V.P., it is kept perpendicular to the V.P. in the initial stage.

Moreover

(i) if the solid has an edge of its base parallel to the H.P. or in the H.P. or on the ground, that edge should be kept perpendicular to the V.P.; if the edge of the base is parallel to the V.P. or in the V.P., it should be kept perpendicular to the H.P.

(ii) If the solid has a corner of its base in the H.P. or on the ground, the sides of the base containing that corner should be kept equally inclined to the V.P.; if the corner is in the V.P., they should be kept equally inclined to the H.P.

(2) Having drawn the projections of the solid in its simple position, the final projections may be obtained by one of the following *two* methods:

(i) Alteration of position: The position of one of the views is altered as required and the other view projected from it.

(ii) Alteration of reference line or auxiliary plane: A new reference line is drawn according to the required conditions, to represent an auxiliary plane and the final view projected on it.

In the first method, the reproduction of a view accurately in the altered position is likely to take considerable time, specially, when the solid has curved surfaces or too many edges and corners. In such cases, it is easier and more convenient to adopt the second method. Sufficient care must however be taken in transferring the distances of various points from their respective reference lines.

After determining the positions of all the points for the corners in the final view, difficulty is often felt in completing the view correctly. The following sequence for joining the corners may be adopted:

(a) Draw the lines for the edges of the visible base. The base, which (compared to the other base) is further away from *xy* in one view, will be fully visible in the other view.

(b) Draw the lines for the longer edges. The lines which pass through the figure of the visible base should be dashed lines.

(c) Draw the lines for the edges of the other base.

It should always be remembered that, when two lines representing the edges cross each other, one of them must be hidden and should therefore be drawn as a dashed line.

13-3-1. AXIS INCLINED TO THE V.P. AND PARALLEL TO THE H.P.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 29 for the following problem.

Problem 13-10. (fig. 13-21): *Draw the projections of a pentagonal prism, base 25 mm side and axis 50 mm long, resting on one of its rectangular faces on the H.P., with the axis inclined at 45° to the V.P.*

In the simple position, assume the prism to be on one of its faces on the ground with the axis perpendicular to the V.P.

Draw the pentagon in the front view with one side in *xy* and project the top view [fig. 13-21(i)].

The shape and size of the figure in the top view will not change, so long as the prism has its face on the H.P. The respective distances of all the corners in the front view from *xy* will also remain constant.

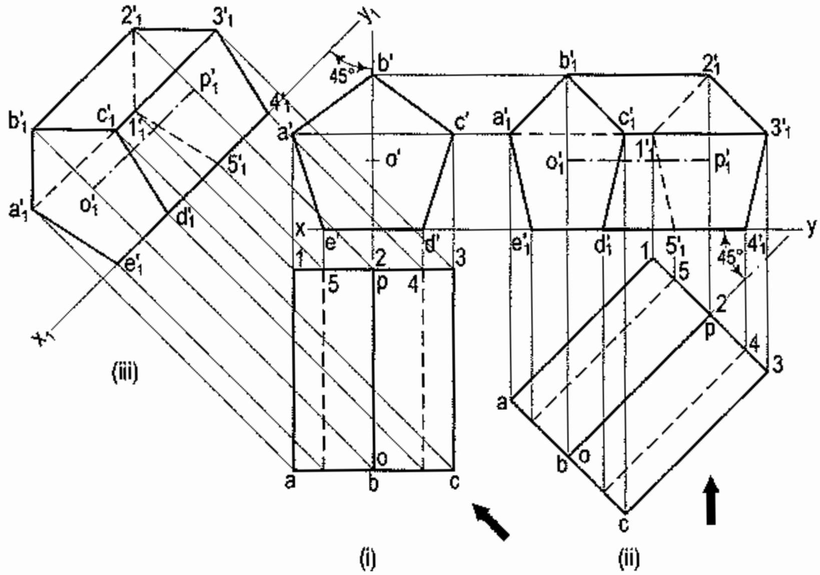


FIG. 13-21

Method I: [fig. 13-21(ii)]:

(i) Alter the position of the top view, i.e. reproduce it so that the axis is inclined at 45° to *xy*. Project all the points upwards from this top view and horizontally from the first front view, e.g. a vertical from *a* intersecting a horizontal from *a'* at a point a'_1.

(ii) Complete the pentagon $a'_1b'_1c'_1d'_1e'_1$ for the fully visible end of the prism. Next, draw the lines for the longer edges and finally, draw the lines for the edges of the other end. Note carefully that the lines $a'_1 1'_1$, $1'_1 2'_1$ and $1'_1 5'_1$ are dashed lines. $e'_1 5'_1$ is also hidden but it coincides with other visible lines.

Method II: [fig. 13-21(iii)]:

(i) Draw a new reference line x_1y_1, making 45° angle with the top view of the axis, to represent an auxiliary vertical plane.

(ii) Draw projectors from all the points in the top view perpendicular to x_1y_1 and on them, mark points keeping the distance of each point from x_1y_1 equal to its distance from *xy* in the front view. Join the points as already explained. The auxiliary front view and the top view are the required projections.

Problem 13-11. (fig. 13-22): *Draw the projections of a cylinder 75 mm diameter and 100 mm long, lying on the ground with its axis inclined at 30° to the V.P. and parallel to the ground.*

Adopt the same methods as in the previous problem. The ellipses for the ends should be joined by common tangents. Note that half of the ellipse for the hidden base will be drawn as dashed line.

Fig. 13-22(iii) shows the front view obtained by the method II.

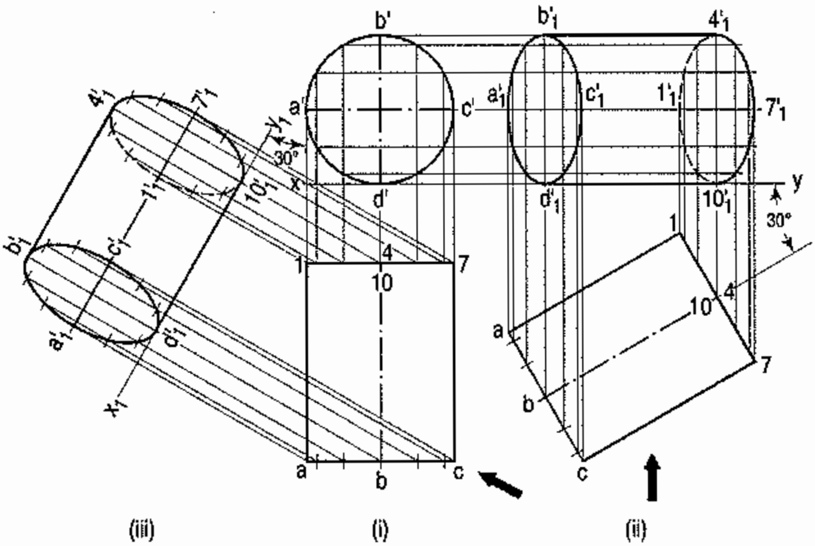


FIG. 13-22

13-3-2. AXIS INCLINED TO THE H.P. AND PARALLEL TO THE V.P.

Problem 13-12. (fig. 13-23): *A hexagonal pyramid, base 25 mm side and axis 50 mm long, has an edge of its base on the ground. Its axis is inclined at 30° to the ground and parallel to the V.P. Draw its projections.*

In the initial position assume the axis to be perpendicular to the H.P.

Draw the projections with the base in *xy* and its one edge perpendicular to the V.P. [fig. 13-23(i)].

If the pyramid is now tilted about the edge *AF* (or *CD*) the axis will become inclined to the H.P. but will remain parallel to the V.P. The distances of all the corners from the V.P. will remain constant.

The front view will not be affected except in its position in relation to *xy*. The new top view will have its corners at same distances from *xy*, as before.

Method I: [fig. 13-23(ii)]:

(i) Reproduce the front view so that the axis makes 30° angle with *xy* and the point *a'* remains in *xy*.

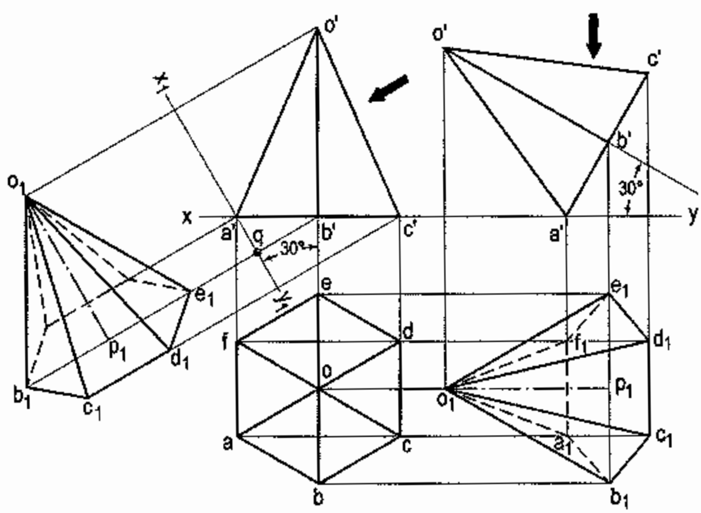
(ii) Project all the points vertically from this front view and horizontally from the first top view. Complete the new top view by drawing (a) lines joining the apex o'_1 with the corners of the base and (b) lines for the edges of the base.

The base will be partly hidden as shown by dashed line a_1b_1, e_1f_1 and f_1a_1. Similarly o_1f_1 and o_1a_1 are also dashed lines.

Method II: [fig. 13-23(iii)]:

(i) Through *a'* draw a new reference line x_1y_1 inclined at 30° to the axis, to represent an auxiliary inclined plane.

(ii) From the front view project the required top view on x_1y_1, keeping the distance of each point from x_1y_1 equal to the distance of its first top view from *xy*, viz. $e_1q = eb'$ etc.



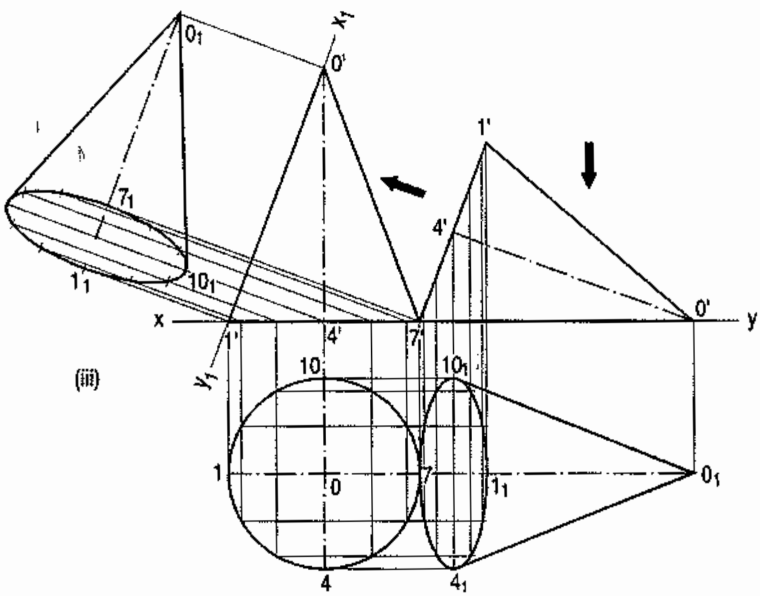
(iii) (i) (ii)

FIG. 13-23

Problem 13-13. (fig. 13-24): *Draw the projections of a cone, base 75 mm diameter and axis 100 mm long, lying on the H.P. on one of its generators with the axis parallel to the V.P.*

(i) Assuming the cone to be resting on its base on the ground, draw its projections.

(ii) Re-draw the front view so that the line $o'7'$ (or $o'1'$) is in xy. Project the required top view as shown. The lines from o_1 should be tangents to the ellipse.



(iii)

(i) (ii)

FIG. 13-24